# Value-Adding Designs and the Efficiency Augmented Solow Growth Model

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**Abstract**

Long-run welfare growth can be achieved only by using finite resources more productively within an economy, but the mechanisms that will ensure this outcome are less understood. Nevertheless, a growing consensus has emerged among growth theorists that an economy can satisfy its perpetual growth agenda by effectively controlling the means of innovation. However, prominent models of endogenous growth have failed to effectively document the process of innovation or long-run productivity growth within an economy and the drivers behind it. This paper will attempt to show that technical productivity consists of measurable components, and that growth of competitive firms' cash reserves and the rate of knowledge spillover among them can explain the variations in long-run economic growth.

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## I. Introduction: Innovation Ensures Continual Economic Growth, But Who Does It and Why?

What determines the long-run growth of nations? Nobel laureate Robert Solow, in an attempt to refute the Harrod-Domar analysis of equilibrium growth, showed that if the output-capital ratio can be deliberately determined, perpetual growth in income per capita through capital accumulation is impossible.[2] Instead, long-run equilibrium growth is determined by a variable called the Solow Residual. In a confession of ignorance, Solow conceded that this variable is exogenous to his model and "may have no apparent explanation at all."[3]

What is the Solow Residual? Most growth theorists agree that it is a measure of Technical Productivity within an economy. However, the effect that this term has on aggregate production is less clear. Technical Productivity is said to augment the productivity of labor and/or capital in a Cobb-Douglas production function, but a consensus on this issue has not been reached. Nevertheless, growth theorists refuse to relinquish the determination of Technical Productivity to random events beyond an economy's control. Prominent models explaining the process of endogenous Technical Productivity growth include works by Shell and, more recently, Romer, Grossman & Helpman and Klepper.

### *Competitive Firms Finance Innovation With Internally-Generated Free Cash Flow*

Shell posited that governments drive innovation by supporting R&D with marginal increases in tax revenues due to improvements in productivity.[4] This intuitively makes sense under perfect competition. Indeed, public subsidies and research grants are often provided to fund innovation within an economy, thus accounting for a positive relationship between public research expenditure and productivity growth (Figure 1).[5]

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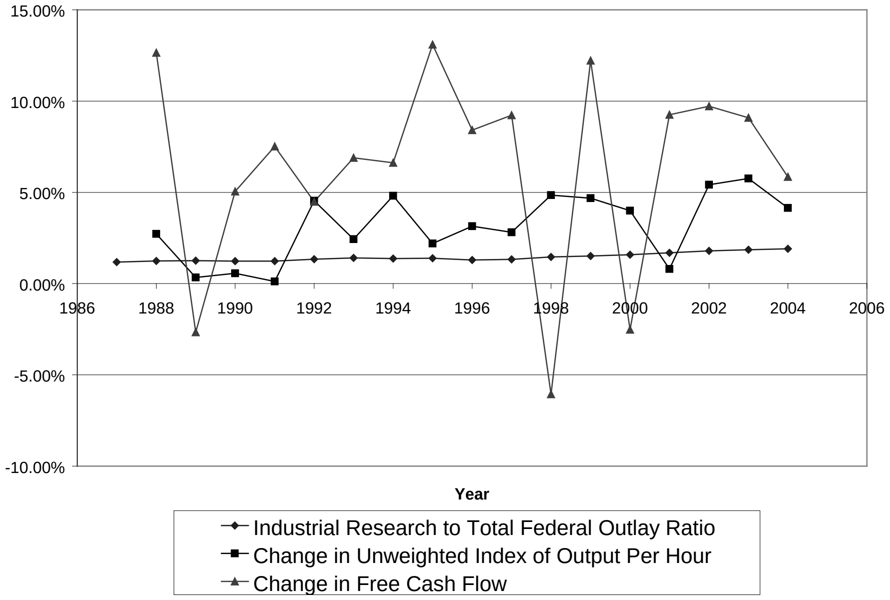
2 Robert M. Solow. "A Contribution to the Theory of Economic Growth." The Quarterly Journal of Economics, Vol. 70, No. 1. (1956): 65-94.

3 Robert M. Solow. "Perspectives on Growth Theory." The Journal of Economic Perspectives, Vol. 8, No. 1. (1994): 45-54.

4 Karl Shell. "A Model of Inventive Activity and Capital Accumulation." In Essays on the Theory of Optimal Economic Growth, edited by Shell, Karl. Cambridge: MIT Press, 1967, 67-85.

5 U.S. Office of Management and Budget, Budget of the United States Government, Historical Tables, annual; U.S. Bureau of Labor Statistics; U.S. Bureau of Economic Analysis.

**Figure 1: Industrial Research to Federal Outlays Ratio and Productivity in NAICS industries**




However, this relationship appears to be weak because consistent, though small, growth in federal industrial research expenditures (consisting of research spending on General Science, Energy, Agriculture, Labor and Health) can hardly explain the variations in average annual productivity growth across 138 NAICS industries, which share a closer relationship with the free cash flow of profit-seeking US firms. The lack of a price mechanism to reflect asymmetric information would explain this weak relationship. Without a mechanism to accurately answer the questions of what, for whom and how much to innovate during a fiscal period, government will be inefficient in allocating resources for innovation and it is unlikely that public provision of technical knowledge permanently drives economic growth.

Therefore, the responsibility of innovation lies in the hands of competitive, profit-maximizing firms. According to Grossman & Helpman,[6],[7] private innovation is financed by capital markets and under perfect information, i.e. lenders are willing to provide credit when innovative outcomes are known. It

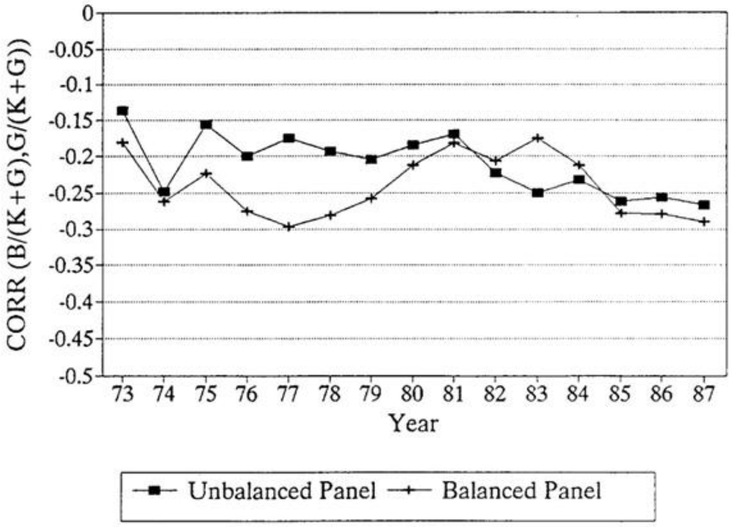
6 Gene M. Grossman and Elhanan Helpman. "Quality Ladders in the Theory of Growth." The Review of Economic Studies Vol. 58, No. 1. (1991): 43-61.

7 Gene M. Grossman and Elhanan Helpman. "Endogenous Innovation in the Theory of Growth." The Journal of Economic Perspectives Vol. 8, No. 1. (1994): 23-44.

should not come as a surprise that relaxing the assumption of perfect information will frustrate this model, especially since R&D projects yield highly uncertain outcomes with dubious benefits in reality. Therefore, a firm's innovative abilities and knowledge are intangible assets that serve as weak collateral, presenting significant risks for investors. As a result, investors demand high rates of return on their investments, making borrowing from capital markets to finance R&D projects prohibitively costly for firms.

Nevertheless, some firms, namely pharmaceutical and biotechnology companies, rely extensively on innovative processes and intangible knowledge for survival and are able to use their patents as collateral for their loans. Here it is important to note that knowledge, while intangible in nature, can indeed be codified and replicated, rendering it tangible in a sense. Patents are the mechanisms through which this transformation takes place since salient details of an innovative process are documented explicitly for replication purposes. To the extent that the patented process produces a product or service that commands a premium over its costs of production, patents can provide predictable cash flows for up to 20 years that can be used as collateral by the debtors. Therefore, for the purpose of this paper, any asset that provides a predictable stream of cash flows can be considered a tangible asset. Intangible assets, such as trademarks, culture, and experience, are characterized by their infinite life spans. We will discuss their distinction in detail later.

**Figure 2: Correlation of Leverage and R&D Capital**



Indeed, evidence for this argument was presented by Hall[8] when she found that R&D intensity of a firm, or the ratio of the economic value of a manufacturing firm's stock of R&D assets ($G$) to net physical and knowledge capital ($K+G$), is negatively correlated with the ratio of debt it currently owes ($B$) to net physical and knowledge capital (Figure 2).[9] This means that an increase in debt to net assets will serve to mainly increase physical and not knowledge assets, hence reducing the R&D intensity of the firm.

One can argue that this market failure can be overcome by venture capitalists, which as a group of specialized investors, routinely finances R&D for outsized returns when they exit their investments through public listings. But despite venture capitalists' expertise in evaluating R&D projects, they finance innovation only to jump start the process and have limited participation in the continual growth of technical productivity, especially in companies that are past the startup phase.

Since financing innovation through the capital markets is prohibitively costly, competitive firms will have to depend on their own cash reserves. Supporting evidence for this view is shown in Figure 1, which reveals an obvious one year lag between changes in internally-generated free cash flow (FCF), which is the cash left over at the end of a fiscal period after tax, investment, liquidity, and productivity growth concerns are met. This observation also reinforces the view that firms fund innovation on hindsight (with accrued cash from the last fiscal period) instead of foresight (forecasting uncertain R&D sales outcome). Accordingly, this paper hypothesizes that the change in cash reserves or FCF drives Technical Productivity growth among firms within an economy.

Against this background, we shall construct a model of endogenous innovation. But before revealing the model, we need to define Technical Productivity in terms of its constituents which form the basis of the model. Thereafter, Section II offers a glimpse of how a firm generates FCF to finance innovation before identifying other factors that will affect a firm's decision to innovate and why competitive firms, both large and small, participate in the process. In Section III, our model of endogenous innovation within an economy will be constructed by aggregating individual firms into a closed, single-industry economy, and its mechanisms will be studied. Lastly, Section IV analyzes the welfare implications of the model and the role of the government.

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8 Bronwyn H. Hall. "Investment and Research and Development at the Firm Level: Does the Source of Financing Matter?" National Bureau of Economic Research Working Paper No. 4096 (1992).

9 Figure 1, Hall 1992.

### *Technical Productivity Consists of Mechanical Efficiency and Value-adding Design*

Thus far, various discussions about the growth of Technical Productivity have failed to provide a measurable definition of the mysterious term, causing scholars to speculate on what it actually represents. To minimize disparities, growth theorists should first agree on how to define Technical Productivity in measurable terms, which will then form a common platform for scholars to discuss and evaluate mechanisms affecting long-run Technical Productivity growth. Therefore, we will begin by breaking down Technical Productivity into its fundamental constituents and suggesting means of measuring them.

Throughout this paper, a two-factor Cobb-Douglas production function with Hicks Neutral Technical Productivity ($A$) will be used. The function is expressed as:

$$Y = A(K)^{\alpha} (L)^{1-\alpha}$$

where output ($Y$) is produced by capital ($K$) and labor ($L$) and is augmented by $A$. In addition, $0 < \alpha < 1$.

The key assumption here is that A consists of two components: *Mechanical Efficiency (E)* and *Value-adding Design (V)*. Assuming that the two components augment capital and labor respectively, the production function becomes

$$Y = (EK)^{\alpha} (VL)^{1-\alpha} \quad (1)$$

and Technical Productivity is

$$A = (E)^{\alpha} (V)^{1-\alpha} \quad (2)$$

In (2), $E$ and $V$ augment capital and labor respectively. To understand why, let us assume that capital represents the stock of unit dollars invested in non-labor factors of production such as energy, machinery, or raw materials. Hence, $E$ is the *ratio of power produced to power purchased per unit capital,* where power is expressed as the amount of work or energy per unit of time. By the *Law of Conservation of Energy,* which states that energy can be neither created nor destroyed, $E$ will be bounded between zero and one: $0 \leq E \leq 1$. Hence, $E$ diminishes capital instead of augmenting it. We can see how this ratio works with the example of a car. In filling up the gas tank of a car, we exchanged money (capital) for 20 gallons of gas (latent energy). Although the car may use up the whole tank, it can never fully utilize every single fluid ounce to power the car for the driver's purpose. Some energy will be wasted in producing unwanted power such as sound, light or heat and the car's $E$ will be less than one but more than zero. Also, suppose labor refers to an able-bodied human and Value-adding Design ($V$) is *a complete set of vocational information used for a value-adding process.* Therefore, $V$ augments labor and not capital because we assume that only humans can invent or revamp a production process and subsequently implement the changes.

In other words, $V$ is simply an intermediate step in a production process. For example, an automotive manufacturing firm will possess at least four $V$'s for a single product: (1) specification design and patent application; (2) chassis construction by welding custom-designed steel pieces together; (3) axles, fuel tank and engine added to the chassis; (4) interior fixing and paint-job. To count as an intermediate step, the end product of each step must be able to fetch a *market price above costs if sold as-is*. For example, an in-house car design can be sold at a market price no different from the work of a professional car designer, and a completed chassis can fetch the same price as a ready-made chassis available at a car mechanic workshop. However, for firms manufacturing fuel-guzzling trucks in a high fuel price environment, the phase during which a fuel-consuming engine is added to the chassis will not count as a design. To make this argument more tractable, let us consider an average complete truck and an average fuel cost environment as our starting point. This truck is average in every sense: average tires, chassis and engine (an average of all engine technology in this world), etcetera. Then, the truck is fixed with a fuel-consuming engine while the price per gallon of gas skyrockets to $4. Because of this setup, the price of this new, but very badly designed, truck will have to suffer a steep discount before it is purchased. If this discount values the truck below its costs, then the process of putting in the new engine and everything associated with its design is worthless. Nevertheless, the truck maker can be profitable again should it revert back to the average engine. Thus, this step of putting in the fuel-consuming engine should not count as a design because it does not add value although prior phases will still count as value-adding designs. Therefore, $V$ is discrete and cumulative within a firm, industry and economy by extension.

To fully understand what a value-adding design is, we need to define *vocational information* as well. A piece of vocational information is a *professional data or fact required in production* that can be *tacit* or *explicit* in nature.[10] *Tacit* vocational information ($I_{tac}$) is difficult to codify and decipher and

10 Jeffrey H. Dyer and Kentaro Nobeoka. "Creating and Sharing a High-Performance Knowledge-Sharing Network: The Toyota Case." Strategic Management Journal Vol. 21, No. 3. (2000): 345-367.

is embodied within labor. For example, the teamwork aptitude of a worker and his thought process under stressful conditions will be classified as tacit vocational information that he alone possesses. In contrast, *explicit* vocational information ($I_{exp}$) can be codified and easily deciphered, and is embodied within tangible products, such as a library book that discusses the process of welding metals or a production prototype built by Toyota. Note that a piece of vocational information is not necessarily productive; it adds value only within the scope of a value-adding production process, which usually involves both tacit and explicit vocational information. For example, the making of a car chassis requires a welder's tacit ability to work within a team of technicians and his explicit welding skills. Therefore, a Value-adding Design ($V$) is the sum of all tacit and explicit vocational information used within the productive process:

$$V = I_{tac} + I_{exp}$$

Because $I_{tac}$ is hard to quantify, as opposed to counting the actual number of manuals or prototypes that represent $I_{exp}$, it is helpful for us to think of both elements of a discrete value-added design as a percentage of daily production time. Suppose a chassis-building team works for ten hours every day. Of these ten hours, they follow the manufacturing manual and prototype for a total of four hours and the rest are spent developing team dynamics. Therefore, $V = 0.6 + 0.4 = 1$, which is then discrete.

Furthermore, vocational information is *non-rivaled* and *partially excludable.*[11] *Non-rivalry* means that the usage of a good does not diminish its value to another user, while *partial excludability* means that the usage of a good can be prevented through ownership for a limited time. $I_{tac}$ is non-rivaled and fully excludable because implicit information of a person, such as the strong work ethics of a seasoned engineer, does not diminish after the person uses it for an activity; yet he can exclude others from using his skills simply because they are hard to codify and decipher and are embodied within him. The same cannot be said of $I_{exp}$. $I_{exp}$ is non-rivaled but partially excludable. The understanding of a paper by one reader does not diminish its value to another. Access to it can be prevented by online subscription requirements, but one can easily travel to a public library to read the paper (if available) for free. Similarly, patents can be considered as $I_{exp}$ since they have limited lifetimes.

Lastly, there also exists an *a priori* design ($V$=1), which is an able-bodied human's innate ability to manufacture basic tools and goods, such as bricks. Making bricks is not rocket science; one can do so by taking a lump of clay

11 Paul Romer. "Endogenous Technological Change." Journal of Political Economy Vol. 98, No. 5, Part 2. (1990): S71-S102.

from a river bank, shaping it into a block and leaving it in the sun to dry. Therefore, an *a priori* design is obtained via evolution and occurs naturally among humans involving explicit vocational information only. The relevance of this design will be discussed in the following section.

## II. Innovation Within a Firm

The competitive model to be constructed within the next two sections operates under a single-industry, closed economy with many competitive firms producing a differentiated producer durable, which is then used as a capital input for the industry. This is not hard to imagine within a brick-making industry since there are many firms producing slightly differentiated bricks. These products are then used to construct ovens, which are the only productive "machinery," which are in turn combined with labor to produce more bricks, and the cycle goes on. Brick-making might be a mundane industry, but it serves as an appropriate simplifying example. Another industry with the same characteristic would be the financial industry. A financial intermediary firm often bundles its loans and mortgages into tradable securities, which are then sold to private investors. However, the financial industry does not produce depreciable durables and is hence not analyzed here.

### *Capital Replacement Depends on Mechanical Efficiency and Affects FCF*

Since a firm finances innovation with its cash reserves, its growth is proportional to its FCF at the end of every fiscal period. According to financial accounting concepts, FCF is the relevant cash flow to a firm at the end of each fiscal period, defined as:

FCF = (1 - tax rate) EBIT + Depreciation - Increases in net working capital - Capital expenditure

where EBIT is earnings before interest and taxes.[12] Translating this into economic terms, FCF is defined as:

$$\begin{aligned} F_{it} &= \pi_{it} - \delta_{it}K_{it} - \Delta K \\ &= (1-\tau)(TR_{it} - w_{it}L_{it} - r_{it}K_{it}) - \delta_{it}K_{it} - \Delta K \end{aligned} \tag{3}$$

where ($F_{it}$) is the FCF of firm *i* in period *t*; $\pi_{it}$ is after tax, depreciation-adjusted profit of the firm; $\tau$ is the corporate tax rate; $TR_{it}$ is actual revenue received in period *t*; $w_{it}L_{it}$ and $r_{it}K_{it}$ capture labor salaries and other non labor operating

12 Robert C. Higgins. *Analysis for Financial Management*. 7th edition, McGraw-Hill, 2003.

costs; and $\delta_{it}$ represents the ratio of parts replacement costs (actual cash spent to replace worn out parts) per dollar in period t, which is assumed to have an inverse relationship with Mechanical Efficiency ($E$):

$$\delta_{it} = 1 - E_{it} \tag{4}$$

The intuition here is that as the efficiency of a machine increases, its parts last longer and the costs of replacing parts fall, causing a decrease in the ratio of parts replacement costs per dollar of capital.

***Growth of Cash Reserves Enables Firms to Innovate in the Face of Competition***

Furthermore, there exists an *a priori* industry, such as brick-making, which can produce output with the *a priori* design described in the previous section such that the minimum marginal productivity of labor ($MPL_{min}$) is equal to the productivity of an able-bodied worker (brick-maker) in that industry. A worker will be paid his minimum productivity along with the value his vocational information brings to his employer. Therefore, the wage due to a worker is

$$w = MPL_{min} + MPI_{tac} + \mu(MPI_{exp}) \tag{5}$$

where $MPI_{tac}$ is the marginal productivity of fully-excludable tacit vocational information embodied within the worker and $\mu(MPI_{exp})$ is a fraction of the marginal productivity of the explicit vocational information the worker generated. The latter is paid out to labor as a bonus in each period because explicit vocational information is partially excludable and is not embodied within labor. This allows a firm to arbitrarily decide what μ is going to be, hence $0 < \mu < 1$. Any value not distributed in discretionary bonuses will therefore contribute to the firm's FCF at the end of each fiscal period. Therefore, excess cash will accrue to a firm only when it is able to derive new designs containing $I_{exp}$.

With the incumbent firm accumulating cash in excess of its required cost of capital, new firms will enter the industry. Let us suppose that the entrant's production process utilizes more $I_{exp}$ and therefore costs less since the firm is able to pay lower wages. As a result, the entrant is able to set a lower price for a slightly differentiated product, capturing market demand and putting downward pressure on incumbent firms' profitability. Because knowledge takes time to spill over from firm to firm (to be discussed later in this section), the entrant is able to retain its competitive advantage. To maintain profit margins, an incumbent firm is faced with three choices:

1. Commission a product-improvement project to drive production costs down and compete on pricing.
2. Commission a product-improvement project to further differentiate its product and generate more inelastic demand for it.
3. Eliminate production steps that are done more efficiently by the new firm and source intermediate products from it instead. Incumbent firms will retain designs that differentiate products and hope that sales volume and profit margins are not too adversely affected.

Of course, an incumbent firm can execute a combination of the above three options based on expected market responses, using its cash reserves to finance any product improvement project. If the incumbent firm succeeds in executing (1), both the firm and the industry as a whole will experience a growth in the productivity of existing designs, although the number of designs remains the same. If the incumbent firm succeeds in executing (2), a growth in V and not productivity will result instead. Hoping for the best in (3) is the worst choice and the uninventive incumbent firm will often elect to close down, with no impact on either the count or productivity of designs within the industry. Therefore, a firm's cash reserves play a vital role in its ability to cope with competition and the growth of designs. In the case of a firm with negative FCF, i.e. spending on physical assets replacement or expanding beyond the entire amount of cash generated from operations during a fiscal period, the firm will have a lower ability to finance a knowledge-expansion project which takes a lower priority since its benefits are highly uncertain. Therefore, a growth in *V* is directly related to the positive growth of cash reserves, meaning that a positive FCF induces a positive change in *V* for a firm, and vice-versa. This hypothesis finds support in Cohen *et al.*[13] which demonstrates that R&D expenditure varies proportionally to a firm's market capitalization, which correlates positively with FCF.

An interesting point is that even though theory predicts a positive correlation between FCF and a growth in *V*, this correlation holds only for positive FCF at an industry level. Systematic cash drains due to a fall in aggregate market demand will cause firms to downsize or even close down, but the survival of the fittest ensures that industrial consolidation will result in a single firm controlling all productive designs, and hence no reduction in the stock of *V*.

13 Wesley M. Cohen, Richard C. Levin, and David C. Mowery. "Firm Size and R & D Intensity: A Re-Examination" The Journal of Industrial Economics, Vol. 35, No. 4, (1987):543-565.

Eventually, this natural monopoly will generate positive FCF since it controls the entire industry demand and growth in *V* will be at least zero.

***Growth in Value-adding Designs Also Depends on Speed of Knowledge Spillover***

In addition to FCF, the willingness to innovate is also affected by the ability of firms to protect what they have created, i.e. the better knowledge can be protected, the more *V* the firm will generate in any given period since it is prohibitively expensive for rivals to fully imitate a better design. Therefore, the change in the stock of *V* can be expressed as:

$$\Delta V \equiv f\,[\ell_{it}, F_{it-1}] \tag{6}$$

where $\ell_{it}$ measures the time it takes for knowledge to spill over to other firms. Thus, $\ell_{it}$ is termed as the *knowledge spillover lag time* and is expressed as a percentage of a year during which the innovating firm protects its stock of designs, and $0 \leq \ell_{it} \leq 1$. An in-depth discussion of this variable follows. $F_{it\text{-}1}$ is the free cash flow to a representative firm from the prior period.

The ability of a firm to protect its knowledge is affected by the effectiveness of patenting a design. If an economy has strict patent laws and trustworthy enforcement, a firm can reasonably expect to recover all its losses resulting from a breach of intellectual property (IP) rights in court. As a result, rival firms will be less willing to imitate designs since penalties are high. Therefore, a firm will be more willing to innovate since substantial benefits of innovation can be reaped. In contrast, an economy with weak patent law structures and enforcement will encounter less growth in *V* since the costs of imitation of designs is low and a firm's knowledge will spill over to its rivals quickly. However, larger increases in the productivity of existing *V* will be possible since the existing stock of *V* is more accessible and the market is rife with competition. Nonetheless, this is not sufficient to ensure long-term economic growth, which is dependent on consistent growth in the stock of *V* and not the growth in productivity of existing designs.

Besides the effectiveness of patent law, a firm's ability to protect its knowledge is also affected by its employees' moral and contractual obligations to their current employer. For example, an employee that is happy with his working environment will be less willing to join a rival firm, resulting in a lower turnover rate for his employer and less spillover of tacit vocational information to rivals. Likewise, the existence of confidentiality clauses within employment contracts will mean fewer spillovers as well.

Lastly, the ability of a firm to protect knowledge is also affected by the nature of the industry in which it resides. If the industry is built around explicit vocational information, such as the search engine industry, patent laws will have to be effective before firms have the ability to protect their knowledge. However, an industry that is built around tacit vocational information, such as the auto-manufacturing industry, will place less emphasis on patents since imitation requires retraining workers by rival firms, which will take relatively more time. Therefore, the nature of the industry weighs the importance of the previous two factors and $\ell_{it}$ can be expressed as:

$$\ell_{it} \equiv \epsilon_{it} \cdot \Omega_{it} + (1-\epsilon_{it}) \cdot (\Phi_{it}), \tag{7}$$

where $\Omega_{it}$ represents the effectiveness of patent laws and is measured by the ratio of historical compensatory damages awarded by a IP infringement lawsuit to the expected net present value of cash flows from a patent, and $0 \leq \Omega_{it} \leq 1$. $\Phi_{it}$ measures labor's moral or contractual obligations to its current employer, and $0 \leq \Phi_{it} \leq 1$, where $\Phi_{it} = 0$ equals minimum obligations and maximum turnover in employees. To put together $\Phi_{it}$, we have to first quantify both moral and contractual obligations. Moral obligations can be effectively measured by employee surveys designed to tease out their level of job satisfaction and cultural attachment. Contractual obligations can be found by surveying the average number of years an employee is contracted to a company and what confidentiality restrictions the contract states. Hence, $\Phi_{it}$ measures the probability of any given employee quitting the company within a year. $\epsilon_{it}$ weighs the previous two factors in importance and is measured by the ratio of explicit vocational information to total vocational information (explicit and tacit) required to produce a unit of output, and $0 < \epsilon_{it} \leq 1$. Given that *a priori* knowledge ($V = 1$) is entirely explicit, **$\epsilon_{it}$ is strictly > 0**. This means that effective patent laws will always play a role in innovation.

## III. Innovation Within an Economy

Before we aggregate innovation at the firm level into a closed, single-industry economy and incorporate the model into the neo-classical Solow growth model, let us explore the limit to continual growth in Mechanical Efficiency ($E$).

By the Law of Conservation of Energy, we know that $E$ is bounded by nature and $0 < E < 1$. A firm might want to set an optimal level ($E^*$) within the given bounds for its products in order to maximize profits across time since replacement demand approaches zero as $E$ approaches 1 (perfect mechanical efficiency and no wear and tear). Because such firms will hesitate to make their products perfectly efficient, they will maintain $E^*$ independently in steady state for optimal profit across time. Proving this assertion will take us too far from the subject at hand and should be a topic for subsequent research. If we assume that $E^*$ indeed exists independently for each firm, then there also exists a weighted average $E^*$ of our single industry economy. The weight used here should be a firm's share of total industrial output.

Now let us incorporate our earlier findings into the neo-classical model of economic growth. Consider the national income accounting identity of a closed real economy:

$$Y = C + I + G$$

where $Y$ is aggregate national income, $C$ is aggregate private consumption, $I$ is aggregate private investment, and $G$ is government expenditure. If we break up $G$ into its consumption and investment components, national income becomes:

$$Y = C_{At} + I_{At} \text{ and } Y - C_{At} = I_{At}$$

where $C_{At}$ and $I_{At}$ are total consumption and investment in period t, respectively.

Since our closed economy consists of three main agent categories (workers, entrepreneurs and government), let us suppose that workers simply deposit what they do not consume in banks with no reserve requirements, government agencies and firms are surplus- and profit-seeking, respectively, and every dime is saved. In addition, suppose the saving rates for all three agent categories are equivalent. The national income equation becomes:

$$\Pi_A + d = sY = I_A \qquad (8)$$

where $\Pi_A$ represents the sum of current fiscal surplus (tax revenue less recurring expenditures) and post-tax private profits, $d$ is the net amount of worker deposits, and $s$ is the national savings rate.

Next, we return to our FCF analysis in Section II. From equation (3) we have:

$$F = \Pi_A - \delta K - \Delta K.$$

Rearranging (3) and combining it with (8), we obtain

$$sY = \delta K + \Delta K + F + d \tag{9}$$

If we define efficient capital per knowledge augmented labor ($K_e$) as

$$K_e = EK/VL \tag{10}$$

then from our Cobb-Douglas production function, or equation (1), we will obtain[14]

$$Y/VL = (EK/VL)^{\alpha} = (K_e)^{\alpha} \tag{11}$$

Taking the rate of change[15] of $K_e$ from equation (10),

$$\frac{\Delta K_e}{K_e} = \frac{\Delta E}{E} + \frac{\Delta K}{K} - \frac{\Delta V}{V} - \frac{\Delta L}{L}$$

which when combined with equation (9) becomes

$$\frac{\Delta K_e}{K_e} = g_E + \frac{sY - \delta K - F - d}{K} - g_V - g_5$$

where $g_E$, $g_V$ and $g_L$ represent the growth rates of Mechanical Efficiency ($E$), Value-adding Designs ($V$) and labor ($L$), respectively. Multiplying the equation throughout with equation (10), change in efficient capital becomes

$$\Delta K_e = s(Y/VL) \cdot E - [(F + d)/K] \cdot K_e - (\delta + g_V + g_L - g_E)\, K_e,$$

and with (11),

$$\Delta K_e = s(K_e)\alpha \;\; \cdot E - [\delta + (F + d)/K + gV + gL - gE]\, K_e, \tag{12}$$

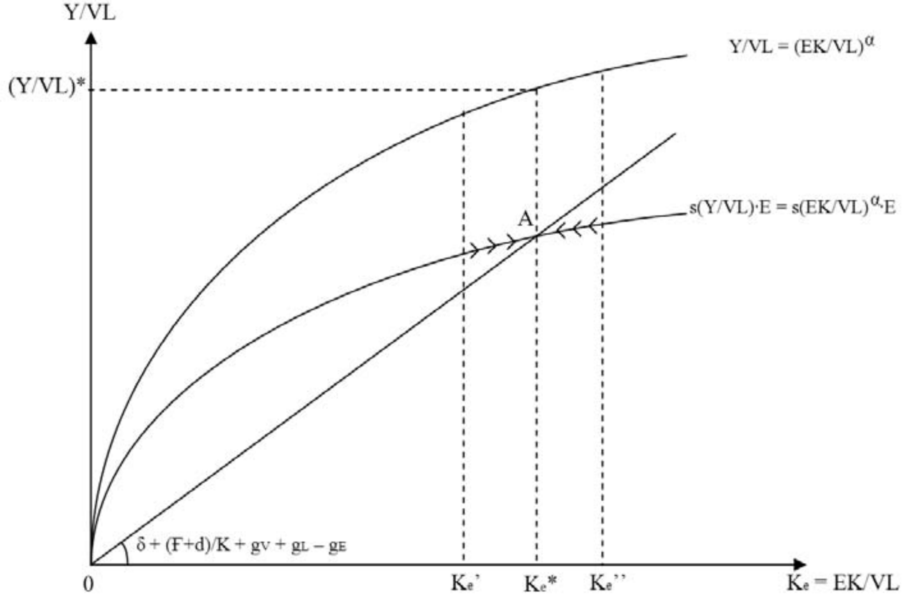
hence yielding the Efficiency Augmented Solow Growth Model.

14 Divide equation (1) by value-adding design augmented labor (*VL*).

15 The rate of change is derived by taking the natural logarithm of equation (14) and then differentiating the result once with respect to time.

Equation (12) shows that a change in $K_e$ equals the difference between efficient savings per knowledge augmented labor and replacement requirements. The latter increases with replacement costs per dollar of capital ($\delta$), total free cash ($F$+$d$) per dollar of capital, growth in Value-adding Designs ($V$) and labor ($L$), but diminishes with a growth in Mechanical Efficiency ($E$). In addition, a point should be made about the relationship between deposits ($d$) and replacement requirements. Suppose firms borrow from capital markets to finance capital expenditures and receivables. This allows firms to use more profits for innovation purposes, and FCF and $V$ will correspondingly rise. Therefore, a rise in $d$ increases replacement requirements. Graphically, equations (11) and (12) can be shown below:

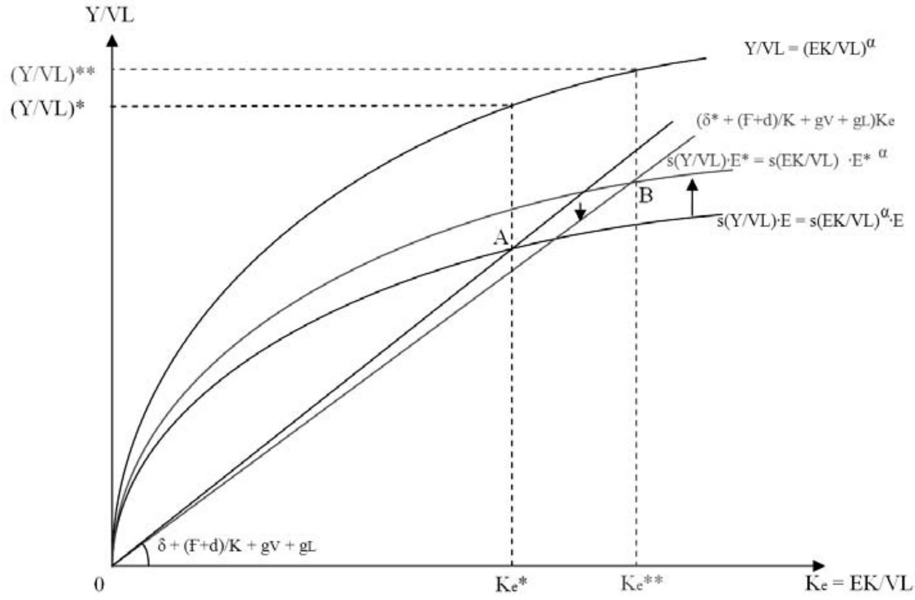
**Figure 3: Efficiency Augmented Solow Growth Model**




In Figure 3, $K_e$' denotes a level of efficient capital per knowledge augmented labor that has a lower replacement requirement than available savings per knowledge augmented labor. Ceteris paribus, an economy currently at $K_e$' will experience an increase in its stock of capital ($K$) and hence a rise in $K_e$. However, if the economy is currently at $K_e$", available savings are insufficient to cover replacement requirements and firms will substitute labor for capital, causing a fall in $K_e$. Eventually, a one-period equilibrium ($A$) is reached, where $K_e$ remains unchanged and the economy attains a fixed real income per knowledge augmented labor ratio. So far, the above analysis did nothing more than review Solow's 1956 insight that if the output-capital ratio is endogenous, and labor and capital are substitutable, then explosive economic growth attained through capital accumulation will eventually peter out.

Suppose now that the previous economy has not attained its optimal level of Mechanical Efficiency ($E^*$) and that its current level of efficiency is lower than the optimal level, i.e. $E < E^*$ and $g_E$ is zero. If all firms within the economy optimize the mechanical efficiency of their products and operate under optimality forever, then the economy will experience a rise in $K_e$ since the savings component according to equation (12) increases while the replacement component decreases since growth in Mechanical Efficiency ($g_E$) is positive and replacement costs per dollar of capital ($\delta = 1\text{-}E$) decreases. Using equation (11), the ratio of income per knowledge augmented labor ($Y/VL$) will correspondingly rise. However, once the economy attains optimal efficiency, $g_E$ will become zero and replacement requirements will rise permanently. Hence, $K_e$ will fall and $Y/VL$ will adjust downwards. Nonetheless, a permanent fall in replacement costs ($\delta$ to $\delta^*$) results in a higher level of $Y/VL$. Graphically, the changes are shown in Figure 4, with the economy attaining a new one period equilibrium *B*.

**Figure 4: Steady State with Optimal Efficiency**



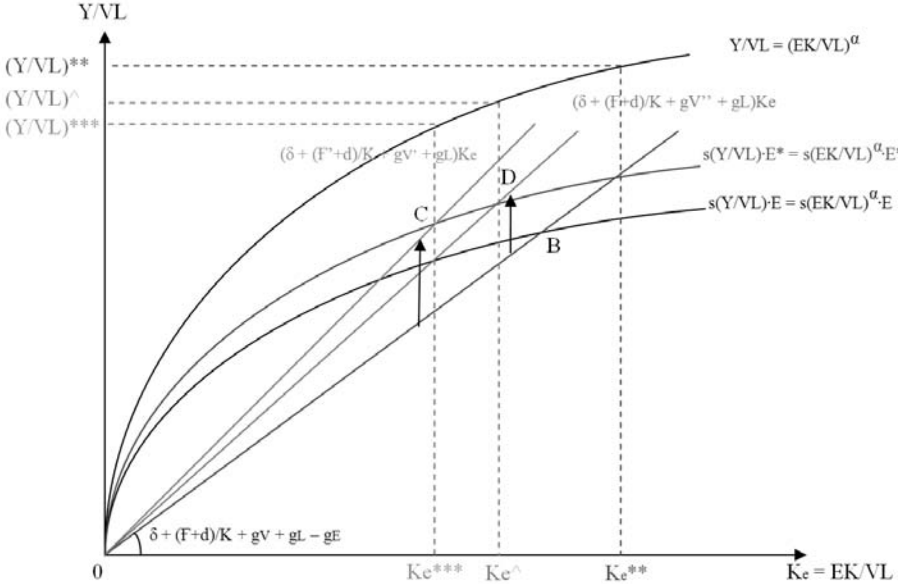
Further, this increase in $E$ to $E^*$ will increase revenue and reduce parts replacement costs of a firm, therefore permanently increasing FCF (from $F$ to $F'$). According to equation (6), the economy will experience a rise in its growth rate of value-adding designs (from $g_V$ to $g_V'$). Subsequently, equation (12) shows that the replacement requirement will increase via a rise in net free cash per dollar capital $(F+d)/K$ and $g_V$. With the savings component remaining unchanged, $K_e$ will fall, causing $Y/VL$ to fall. This result occurs because, ceteris paribus, there is more spare cash to spend on innovation, causing the stock of $V$ to increase faster than other factors of production. Consequently, our closed economy finally attains its steady state $C$, as shown graphically in Figure 5.

Finally, let us suppose there is a permanent improvement in the legislative landscape such that patent laws are more effectively enforced. Better patent laws ($\Omega_{it}$) cause knowledge spillover lag time ($\ell_{it}$) to rise according to equation (7):

$$\ell_{it} \equiv \epsilon_{it} \cdot \Omega_{it} + (1-\epsilon_{it}) \cdot (\Phi_{it}).$$

Using equation (6) once more, the economy will experience a rise in $g_v$ (from $g_V$ to $g_V''$) without a corresponding rise in FCF. Even though replacement requirement increases as per our previous case, the ray from origin experiences a smaller rise, causing smaller decreases in $K_e$ and $Y/VL$ and a change in steady state from $B$ to $D$. This result indicates that an increase in knowledge spillover lag time has a smaller effect on the growth of $V$ than an increase in FCF, and is shown in Figure 5.

**Figure 5: Steady States under Optimal Efficiency with Variou Growth Rates of V**




Now that we have reviewed the mechanisms of the model, we shall move on to explore the effects of endogenous innovation on long term economic welfare or the growth rate of real income per capita within an economy. First, let us assume that the long-run ratio of labor force to population is constant, with the growth rate of population equaling the growth rate of the labor force. Therefore, the growth rate of income per worker will equate to the growth rate of income per capita. In addition, from equation (11) we know that

$$Y/VL = (EK/VL)^{\alpha} = (Ke)^{\alpha}$$

which gives

$$Y/L = (Ke)^{\alpha} \cdot V.$$

Taking its rate of change,[16] we obtain

$$g_{Y/L} = \alpha \cdot g_{Ke} + g_V$$

where $g_{Y/L}$, $g_{Ke}$ and $g_V$ are the growth rates of real income per capita, efficient

16 The rate of change is derived by taking the natural logarithm of the previous equation, and then differentiating the result once with respect to time.

capital per knowledge augmented labor ($K_e$), and Value-adding Designs ($V$), respectively. Using equation (12) we know that the change in the ratio $K_e$ is zero in steady state, hence

$$g_{Y/L} = g_V.$$

Therefore, the sole determinant of long-term welfare growth is the growth rate of Value-adding Designs. This result is perhaps not surprising, considering the fact that nature put a cap on efficiency growth in the first place. However, what is surprising is that growth in the productivity of existing $V$ does not determine long-run growth at all.

In addition, this result serves to reinforce Solow's insight that long-run growth is determined by growth in the Solow Residual, which we have defined earlier as Technical Productivity ($A$). To prove that this model supports Solow's earlier result, we revisit equation (2), which states that

$$A = (E)^{\alpha} \cdot (V)^{1-\alpha}.$$

By taking the rate of change of the previous equation, we have

$$g_A = \alpha \cdot g_E + (1- \alpha) \cdot g_V$$

where $g_A$ is the growth rate of $A$. Since $g_E$ equals to zero in steady state, $g_A$ is determined solely by $g_V$. This proves that, at least on theoretical grounds, our model of endogenous innovation fits well with Solow's results.

## IV. Concluding Remarks

This paper has shown that long-run growth is achievable by ensuring consistent growth in Value-adding Designs which consist of explicit and implicit vocational information. By creating a pro-competition, pro-intellectual property, and pro-contract enforcement environment, economies can maintain consistent long-run growth in designs and thus attain their objective of perpetual economic prosperity.

However, the model discussed in this paper is constructed with strict assumptions and therefore cannot comprehensively document the innovation process within an economy. Thus, subsequent research should focus on relaxing these assumptions to explore their corresponding effects on long-run growth. To promote discussion in the aspect, I will list three main assumptions that will grate a little with readers.

First of all, Mechanical Efficiency is not the only source of efficiency within an economy since there are at least two other existing sources – Digital and Monetary Efficiency. Digital Efficiency measures the efficiency of digital equipment such as computer processing units, whose output is not power per se, but rather their calculation frequency per unit of time. Monetary Efficiency measures the efficiency of the financial sector within an economy or the ratio of actual investment received by intended targets to initial investment amount. The main problem here lies in the applicability of the Law of Conservation of Energy to non-manufacturing industries since efficiency might not be bounded between zero and one. Even if there are bounds to Digital and Monetary Efficiency, the former needs to be standardized in line with the other two. Currently, Digital Efficiency is measured by floating point operations per unit of power[17] while Mechanical and Monetary Efficiency are expressed as ratios without units. If the measurement of Digital Efficiency can be standardized without a unit, we will be able to derive an aggregate efficiency ratio.

In addition, the assumption that firms utilize their own durable products for production is not realistic. This means that replacement costs per capital for a firm's machinery are different from that of the durable that it produces. A case in point is consumer products manufacturing, where final goods are not used for production but consumption. The possible solution for this would be to extend the model discussed here to the entire supply chain of an industry.

Lastly, the deliberate control an economy has over its innovation process is diluted by random factors such as the nature of an industry ($\epsilon_{it}$) which weighs the patent laws, and workers' moral and contractual obligations to current employers in importance. This is a factor that evolves over time with no apparent room for deliberate control. Additionally, the deliberate control an economy has over $E$ is ambiguous. One can conjecture that changes in $E$ are a direct result of changes in $V$, but evidence for this is not concrete at the time of writing.

Despite these limitations, the model discussed within this paper forms a basis for us to understand endogenous innovation and can be used to identify controllable variables for both private and public agents.

17 Sushant Sharma, Chung-Hsing Hsu, and Wu-chun Feng. "Making a Case for a Green500 List." The Green500 List 01 March 2007 <http://www.green500.org/Home.html>.

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